Example of the written math test

Tasks

1) In the set of all real numbers solve the equation
$\sqrt{3x+10} - \sqrt{x+4} = 2$
and find the conditions of solvability.

2) In the set of all real numbers solve the equation
$\log(x+13) - \log(x-3) = \log(x+4)$
and find the conditions of solvability.

3) In the set of all real numbers solve the inequality
$\frac{3x+1}{4x-3} \leq 1.$

4) In the set of all real numbers solve the equation
$2 \cdot \sin^2 x + \sqrt{3} \cdot \sin x = 0.$

5) Find the point P which is the intersection of lines $x + 3y + 2 = 0$ and $2x - 3y - 5 = 0$. Write the equation of the line that passes through this point P and is orthogonal to the line $5x - 3y + 1 = 0$.

Each task is evaluated by 0-6 points according to the degree of completion.

Solution

1)

$$\begin{aligned} \sqrt{3x+10} - \sqrt{x+4} &= 2 \\ \sqrt{3x+10} &= 2 + \sqrt{x+4} \\ 3x + 10 &= 4 + 4\sqrt{x+4} + x + 4 \\ 2x + 2 &= 4\sqrt{x+4} \\ x + 1 &= 2\sqrt{x+4} \\ x^2 + 2x + 1 &= 4(x+4) \\ x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \\ x_1 = 5, x_2 &= -3 \end{aligned}$$

$$3x + 10 \geq 0$$
$$x + 4 \geq 0$$

Proof:

$x_1 = 5$
$L_1 = \sqrt{15+10} - \sqrt{5+4} = 2$
$P_1 = 2$
$L_1 = P_1$

$x_2 = -3$
$L_2 = \sqrt{-9+10} - \sqrt{-3+4} = 0$
$P_2 = 2$
$\mathrm{L}_2 \neq P_2$

Result: $x = 5$

2)

$$\log{(x+13)} - \log(x-3) = \log(x+4) \qquad x+13>0$$

$$\log\frac{x+13}{x-3} = \log{(x+4)} \qquad x-3>0$$

$$x+13 = (x+4)(x-3) \qquad x+4>0$$

$$x+13 = x^2+x-12 \qquad \text{i.e.}$$

$$x^2-25=0 \qquad x>3$$

$$x_{1,2} = \pm 5$$

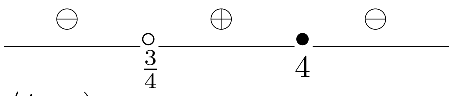
Result: $x = 5$

3)

$$\frac{3x+1}{4x-3} \leq 1$$

$$\frac{3x+1-4x+3}{4x-3} \leq 0$$

$$\frac{4-x}{4x-3} \leq 0$$



Result: $x \in (-\infty, \frac{3}{4}) \cup \langle 4, \infty)$

4)

$$2 \cdot \sin^2 x + \sqrt{3} \cdot \sin x = 0$$

$$\sin x(2\,\sin x + \sqrt{3}) = 0$$

$$\sin x = 0 \qquad\qquad 2\,\sin x + \sqrt{3} = 0$$

$$x_1 = k\pi \qquad\qquad \sin x = -\frac{\sqrt{3}}{2}$$

$$x_2 = \frac{4}{3}\pi + 2k\pi$$

$$x_3 = \frac{5}{3}\pi + 2k\pi$$

Result: $x_1 = k\pi$, $x_2 = \frac{4}{3}\pi + 2k\pi$, $x_3 = \frac{5}{3}\pi + 2k\pi$, $k \in Z$.

5)

$$x+3y+2=0 \qquad\qquad 5x-3y+1=0$$

$$2x-3y-5=0$$

$$\overline{\qquad\qquad\qquad} \qquad\qquad 3x+5y+c=0$$

$$3x=3 \qquad\qquad 3-5=-c$$

$$x=1 \qquad\qquad c=2$$

$$3y+3=0$$

$$y=-1$$

Result: $P = [1, -1]$, $3x+5y+2=0$.